CENTROID THEORY OF TRANSVERSE ELECTRON-PROTON TWO-STREAM INSTABILITY IN A LONG PROTON BUNCH REVISITED: AN ERRATUM

Tai-Sen F. Wang, Paul J. Channell, Robert J. Macek

Los Alamos National Laboratory

Ronald C. Davidson

Princeton Plasma Physics Laboratory

ICFA ECLOUD'04

April 19 - 23, 2004, Napa

Abstract

We have identified a few errors in our recent paper [Phys. Rev. ST Accel. Beams 6, 014204 (2003), Centroid theory of transverse electron-proton two-stream instability in a long proton bunch]. Although the over all qualitative results in the original paper are still correct, an error in Eq. (42) does cause some minor changes in the quantitative results including three figures and alternations in many equations. The most significant change is the required frequency spread for stability given in Eq. (68), which is now found to be twice that given in the original paper. An erratum will be published soon.

In the original paper
[Phys. Rev. ST Accel. Beams **6**, 014204 (2003)]**:**

- We derived the equations of motion for the centroids of the proton bunch and the electron cloud.
  - Damping was included by considering the transverse oscillation frequency distributions of particles.
  - Neglected synchrotron oscillation of protons, axial motion of electrons, and impedance.
- Found the frequency spreads damp the motion of centroids.
  - Damping exponent:
    linear in time for the Lorentzian distribution,
    quadratic in time for the Gaussian distribution.
- Formulated an analytical approach to derive the time-domain asymptotic solution of the linear centroid equations (Lorentzian frequency distribution).
  - Studied the stability of a proton bunch of nonuniform line density propagating through a stationary electron background (one pass interaction).

- Discussed the growth rate of the *e-p* instability and derived a dispersion relation for the case of a uniform density electron background for frequency-domain studies.

  - The growth rate is a function of both space and time.

  - Asymptotic growth rate ($t >>$ growth time):

$$\Gamma(z', t) \approx -\Delta_p + \sqrt{\frac{\omega_\beta \mathcal{J}(z')}{2(t - z'/v)}} \ , \qquad (67)$$

    where $\mathcal{J}(z')$, defined in Eq. (44), characterizes the coupling between the protons and the electrons.

  - For a Lorentzian distribution of proton oscillation frequencies, the instability eventually damps for sufficiently long times $[t > (z'/v) + \omega_\beta \mathcal{J}(z')/(2\Delta_p^2)]$.

  - The threshold for stability was found as

$$\left(\frac{\Delta_p}{\omega_\beta}\right)_t \approx \mathrm{Max}\left\{ \frac{\mathcal{J}(z')}{4} \left[\frac{1 + \mathcal{J}^2(z')/48}{1 + \mathcal{J}^2(z')/32}\right] \right\} , \qquad (68)$$

    where $\mathrm{Max}\{f(z')\} = \mathrm{max.\ of}\ f(z')$.

- Equation (68) is good only for the "e-p mode" which has a wavelength proportional to the electron bounce frequency [the solution of Eq. (28)].

- Threshold and initial growth rate depend on initial conditions; therefore can be quite different for "non-e-p modes".

- The electron oscillation frequency spread causes spatial damping but no temporal damping in the beam frame.

- The asymptotic amplitude ratio between the proton oscillation and the electron oscillation is independent of the frequency spreads.

■ Examples with numerical results were presented by considering proton line densities with uniform and parabolic profiles.

- For the parabolic proton line density, the "e-p modes" are parabolic cylinder functions.

- Equation (68) overestimates the required frequency spread for stability.

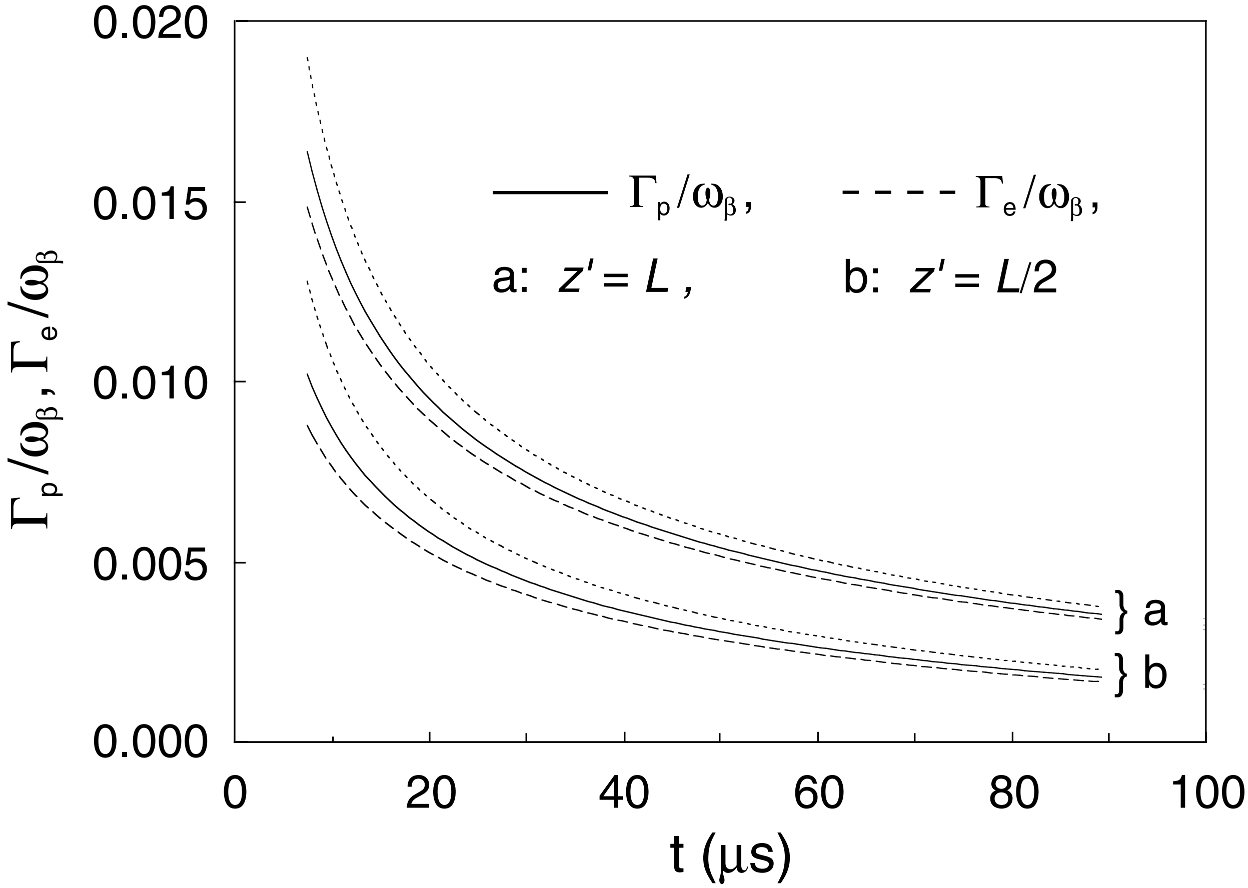


FIG. 3. The growth rates at the tail ($z' = L$) and the center ($z' = L/2$) of the proton bunch are shown as functions of time. Here, Γ_p and Γ_e have been normalized to the betatron frequency ω_β. The solid curve and the dashed curve are computed according to Eqs. (65) and (66), respectively. The dotted curve is computed using the asymptotic approximation in Eq. (67). The parameter values are described in the text.

Revisions to the paper:

- In Eqs. (34), (36) and (38), the equal sign (=) should be replaced by the approximate sign (≈) to indicate the omission of the initial conditions.

- The solution given in Eq. (42) should read

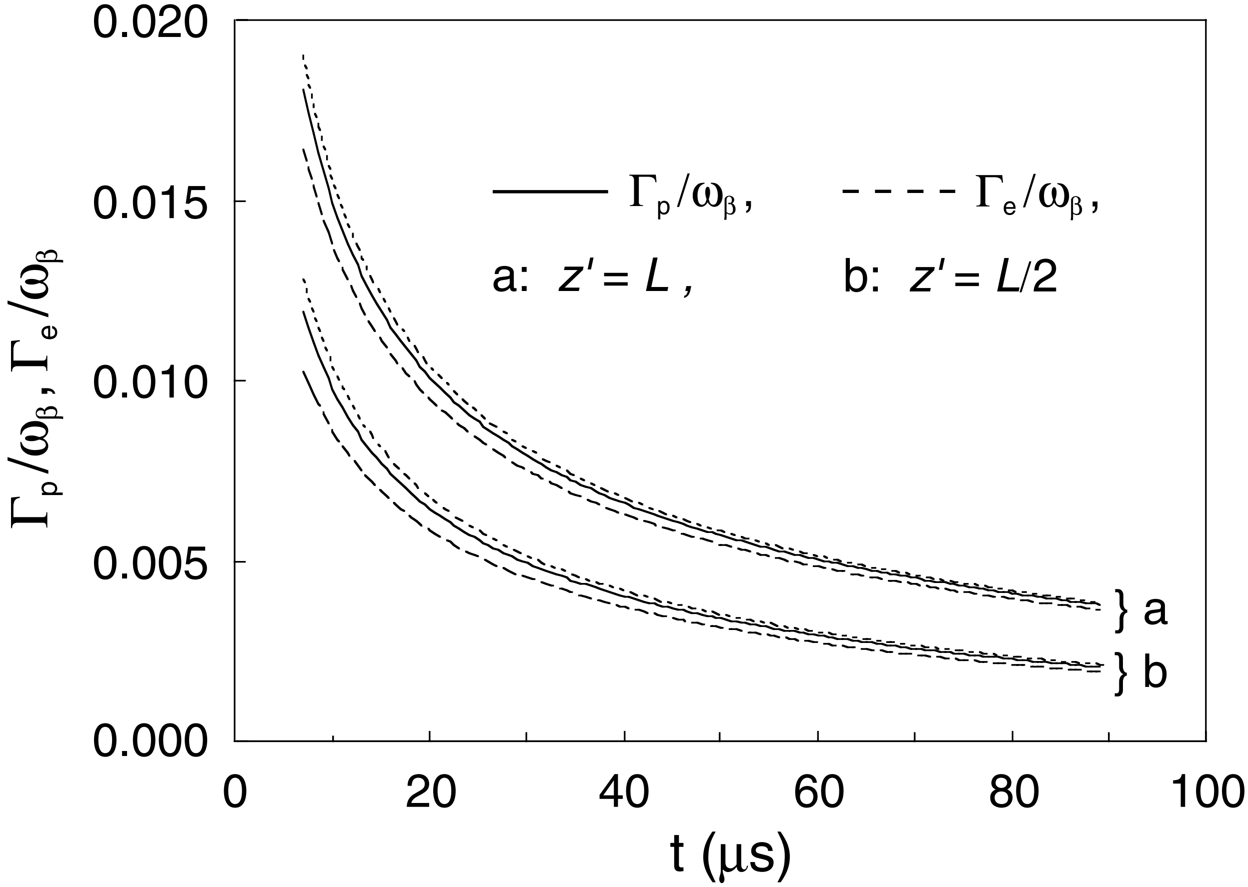
$$\zeta(z', \omega) \approx \left(\frac{1}{\omega^2 - \omega_\beta^2}\right) \exp\left[\frac{i\omega_\beta^2 \mathcal{J}(z')}{\omega^2 - \omega_\beta^2}\right] , \qquad (42)$$

where the factor $1/(\omega^2 - \omega_\beta^2)$ before the exponential function is needed to yield the correct solution for Y_p.

  - This change causes revisions in many equations and changes to Figs. 3, 4 and 5.

- The threshold is now given by

$$\left(\frac{\Delta_p}{\omega_\beta}\right)_t \approx \mathrm{Max}\left\{\frac{\mathcal{J}(z')}{2}\left[\frac{1 + \mathcal{J}^2(z')/32}{1 + \mathcal{J}^2(z')/16}\right]\right\} . \qquad (68)$$



The revised FIG. 3.

The quantity $\mathcal{J}(z')$, defined in Eq. (44), can be rewritten as

$$\mathcal{J}(z') \approx \frac{4i(c/a)^4 r_p r_e}{\gamma(v\omega_\beta)^2 W} \int_0^{z'} \lambda_p(x)\lambda_e(x)\Phi(x)\Psi(x)dx \ ,$$

where $\Phi(x) = R(x)\mathrm{e}^{i\Theta(x)}$ and $\Psi(x) = R(x)\mathrm{e}^{-i\Theta(x)}$ are the two linear independent solutions of

$$\frac{d^2 Y_{en}}{dt^2} + \Omega^2[v(t-t_e)]Y_{en} = 0 \ , \qquad (28)$$

$i = \sqrt{-1}$,

W = Wronskian of $\Phi(x)$ and $\Psi(x)$,

a = proton beam radius.

$\lambda_p(z)$ and $\lambda_e(z)$ = proton and electron line densities,

r_p and r_e = classical proton and electron radii,

$Y_{en} = \mathrm{e}^{\Delta_e t} \times$ (electron centroid displacement),

and

$\Omega(z) = (c/a)\sqrt{2r_e\lambda_p(z)[1-f(z)]} \approx (c/a)\sqrt{2r_e\lambda_p(z)}$.